Computational Mathematics

Taylor Series Representation

Let we consider an ordinary differential equation of first order and first degree like:

With initial condition . This represents

Differentiating w.r.t. x we get from (1)

Differentiating successively, we get:

* Putting the initial condition at then

We get

Now the Taylor’s series of y(x) at is [the Taylor series]

Putting and hence we get from (3)

At

Where,

If the values of are known, then (4) gives a series for.

Once is known, we can compute from (1), (2) and so on.

∴ Now if y be expanded in a Taylor’s series at then:

At

Similarly,

………………………………………………………….

………………………………………………………….

Which are the required Taylor series representation of Differential equation.

**Ex-1:** Using Taylor’s series method, find y(0.1) and y(0.2) by solving with the initial condition y(0)=1.

**Solution:**

1. We know the Taylor series for ordinary differential equation is:

Now we have given:

Now from (1)

1. By Taylor’s series method, we have

Now we have given:

Now from (2)

[Try for the interval (0.04) using two subintervals of size 0.2. with y(0)=0]

**Ex-2:** Using Taylor series method, find y(0.2) and y(0.4) correct to four decimal places by solving With initial condition y(0)=0.

**Hints:**

**Ex-3:** Evaluate y(0.2) correct to six decimal places by Taylor’s series method if y(x) satisfies: with y(0)=0.

**Hints:**

**Ex-4:** Solve: with y(0)=2. [x0=0; y0=2]

Find

(i) y (0.1) ⇾h=0.1

(ii) y (0.2) ⇾h=0.2-0.1=0.1

(iii) y (0.3)⇾h=0.3-0.1=0.2